EQ2415 – Machine Learning and Data Science
HT22

Tutorial 7 A. Honoré, A. Ghosh

1 Graphical models

1.1 Undirected models

We consider the iterated conditional modes (ICM) algorithm for obtaining a local-maximum of the joint distribution defined using a Markov random field. The joint distribution is written as a product of non-negative potential functions ψ_C, and each such function is defined using an energy function E on the nodes of the Markov random field.

Question 1. (a) Write down the energy function for an image denoising model.
(b) Write down a pseudocode for solving the image denoising problem using ICM. One such term in the energy function involves doing pairwise product between the unobserved pixels x_i, x_j. What would be your approach to finding the terms necessary for the summation?
(c) (Bishop 8.13)

1.2 Inference in Graphical models

1.2.1 Inference on a chain

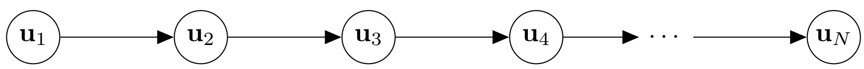

Figure 1: Graphical model of a Markov chain.

Question 2. Consider the undirected version of the graphical model in Fig. 1, where assume that the variables obey a Gaussian distribution.
(a) What is a computational complexity for a naive evaluation to get the marginal distribution of a node $\mathbf{u}_n (1 \leq n \leq N)$?
(b) What will be the cliques in the graph? Having identified the cliques, write the expression for the joint distribution of the graph.
(c) What will be the marginal distribution of a node say $\mathbf{u}_n (1 \leq n \leq N)$ using the above structure of the undirected model? Derive the expression by using the recursion of the message passing algorithm. What's the improvement in terms of computations?
(d) What will be the marginal distribution for a pair of nodes $\mathbf{u}_{n-1}, \mathbf{u}_n (1 \leq n \leq N)$?

Question 3. (Bishop 8.17) Consider the same undirected chain in Q1, except that here consider N = 5, the nodes obey a multinomial distribution, and that nodes $\mathbf{u}_3$ and $\mathbf{u}_5$ are observed.
(a) What is the d-separation property for undirected models?
(b) Use d-separation to show that $\mathbf{u}_2 \perp\!\!\!\perp \mathbf{u}_5 | \mathbf{u}_3$. Show that if message passing algorithm is applied to the evaluation of $p(\mathbf{u}_2|\mathbf{u}_5, \mathbf{u}_3)$, then the result is independent of $\mathbf{u}_5$.

1.3 Sum-product algorithm

Question 4. The sum-product algorithm is perhaps one of the most important inference algorithms discussed in the chapter and perhaps in the book.
(a) What are the three types of messages of interest exchanged during a sum-product algorithm?
(b) (Bishop 8.19) Apply the sum-product algorithm to the graphical model in Fig. 1. Compare your results with part Q1.(c).