

# EQ2415 – Machine Learning and Data Science HT22

Tutorial 6

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## Graphical models in a Bayesian framework

### 1 Bayesian networks

**Question 1.** (a) Bishop 8.1 [1]

**Question 1.** (b) Bishop 8.3. Also, draw the graphical model by first showing that  $p(a, b, c) = p(a)p(c|a)p(b|c)$ .

### 2 Conditional independence

**Question 2.** Show both graphically and analytically if  $x_1, x_2, \dots, x_{t-1} \perp\!\!\!\perp x_{t+1}, x_{t+2}, \dots, x_T | x_t, y_t$ , where  $y_t$  is assumed to depend only on  $x_t$ , then  $x_1, x_2, \dots, x_{t-1} \perp\!\!\!\perp x_{t+2}, x_{t+3}, \dots, x_T | x_t, y_t$ .

**Question 3.** A popular architecture recent architecture that can be represented as a graphical model is a dynamical variational autoencoder (DVAE) [2]. It has been shown to perform reasonably well when applied to signal generation and modeling tasks. Here below is the graphical model of a dynamical VAE called deep Kalman filter (DKF): In the inference mode, this becomes

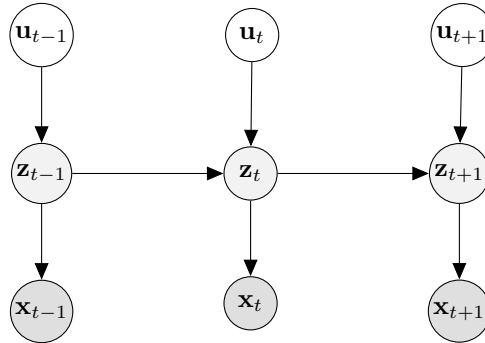


Figure 1: Graphical model of the DKF in generation mode, displayed within 3 states of  $\mathbf{z}_{t-1}$ . The arrows signify dependence between random variables in the graphical model.

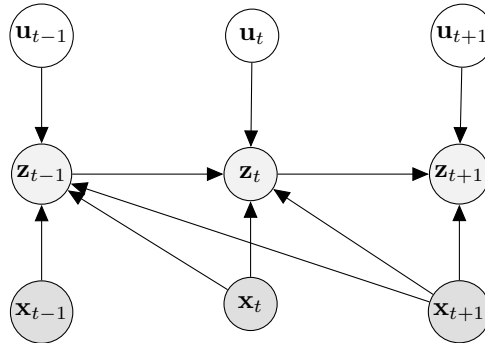


Figure 2: Graphical model of the DKF in inference mode, displayed within 3 states of  $\mathbf{z}_{t-1}$ . The arrows signify dependence between random variables in the graphical model.

(a) Write down the joint distribution of  $p(\mathbf{x}_{1:T}, \mathbf{z}_{1:T} | \mathbf{u}_{1:T})$  for the generation mode in simplified form using D-separation.

(b) Write down the joint distribution  $q(\mathbf{z}_{1:T} | \mathbf{x}_{1:T}, \mathbf{u}_{1:T})$  for the inference mode in simplified form using D-separation.

### 3 Markov random fields

**Question 5.** (a) Bishop 8.12

**Question 5.** (b) Bishop 8.13

### References

- [1] Bishop, Christopher M., and Nasser M. Nasrabadi. Pattern recognition and machine learning. Vol. 4. No. 4. New York: springer, 2006.
- [2] Girin, Laurent, et al. "Dynamical Variational Autoencoders: A Comprehensive Review." Foundations and Trends in Machine Learning 15.1-2 (2021): 1-175.